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AUTOMATIC PLOTTING OF AIRBORNE GEOPHYSICAL SURVEY DATA

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When geophysical measurements are made from the air, it is important to know the position of the aircraft. The position at any time is usually known from a combination of position-fixing and dead-reckoning systems. In-flight optimization of this combination is a well-studied subject. When post-flight track plotting is required for geophysical survey, use can be made of both the initial and terminal position fixes to correct the dead-reckoning track and hence improve on the in-flight version. This technique is applied to compute the track of aircraft conducting ice thickness measurements in the Antarctic, and the effect of certain errors is evaluated. The algebraic results are equally applicable in parts of the world where better navigational aids are available. A relation is derived between the track plotting errors and the errors in the geophysical measurement such that the precision of one is not degraded by errors in the other.

1. Introduction. Many geophysical measurements are possible from an aircraft. These include gravity and magnetic measurements; surface altimetry, infra-red radiometry and no doubt many more.

At the Scott Polar Research Institute, equipment has been developed which can measure the thickness of polar ice sheets by observing a reflected radio wave. Already published are details of the instrument[1] and various field results.[2] The Antarctic flying operations to which this paper mainly refers have been reported.[3] The instrument has been used on the surface and in an aircraft. An aircraft is usually preferred because of the greater ease with which difficult and remote areas can be studied. In the Antarctic Peninsula, single- and twin-engined De Havilland Otter aircraft have been used, and large areas of the inland ice sheet have been sounded using the equipment in Lockheed C121-J Constellation and C130 Hercules aircraft.

Although the navigation systems which have been available were less than ideal, and it is likely that future survey work of this kind will make use of more accurate techniques, yet the general results obtained in this paper are applicable to position-fixing systems and dead-reckoning systems which are more precise than in the examples considered here.

2. Systems used in the Antarctic. An aircraft conducting a geophysical survey carries an instrument or sensor that measures the value of some terrestrial property which varies with geographical position. Radio echo sounding of polar ice sheets measures the time interval between echoes received from ice surface and bottom features. By making certain assumptions about the velocity of propagation and the path of the wave, the thickness of the ice is deduced. In other forms of geophysical survey-

ing—airborne, shipborne and possibly on the ground—the technique and measurand may be different, but we need only assume that some single-valued property is measured with a known resolution.

In some investigations the positions may not be important. For example, the correlation between ice surface elevation and ice thickness can be investigated in the Antarctic without reference to actual position. More usually positions are necessary to describe a measurement completely, and this is the situation which we shall consider.

Position information is usually obtained from a combination of a position-fixing system, by which positions are periodically determined without reference to the dynamic history of the aircraft, and a dead-reckoning system, by which such positions are extrapolated to future time. A consideration of most dead-reckoning and position-fixing systems has already been published.[4] In the Antarctic the problems of maintenance, as well as economics, have limited the navigational systems to the most basic. Track plotting during flight has not been performed to any accuracy greater than that normally used for point to point navigation. Three large stations, McMurdo, Byrd and Amundsen-Scott base at the South Pole, were equipped with TACAN beacons, and these were used by both the C121 and C130 aircraft. Unfortunately the range of these facilities is only about 100 km. at low altitudes. At higher altitudes the range is greater, but the error in position due to bearing errors may be as much as 20 km. at a range of 200 km. Flights from these stations frequently ranged out to 1500 km. and for this reason TACAN was only useful as a terminal navigational aid. Apart from TACAN no radio navigational aid was available. Often the only method of position fixing was visual identification or (what amounts to the same thing) identification on the aircraft radar or later identification on trimetrogon photographs. These methods required the region to be mapped in advance and in many areas of the Antarctic conspicuous surface features are absent. The aircraft radar extended the range of visual identification, but with a correspondingly greater error. Within visual range of mapped surface features the trimetrogon cameras were regularly used to permit later resection of position (Fig. 1). In one method, we have used a semi-automatic digital coordinate reader to measure the positions of known points on oblique aerial photographs. The angles subtended by such control points could then be computed (Fig. 2).

Even at 100 km. range the error due to the Earth's curvature is only about 1° and therefore the largest error is usually due to the different attitudes of the aircraft and the control points. A difference from the horizontal of 5° would introduce an error of 0·5 per cent in the calculated angle subtended by the control points.

Subtended angles have often been measured with a perspective overlay. On a horizontal surface one may construct radials from the nadir, and project these into the focal plane of the photograph. Vertical planes passing through the observer's vertical intersect the plane of the photo-

FIG. 1. An oblique aerial photograph. A resected position is obtained using angles determined from the photograph (U.S. Navy)

graph in straight lines, therefore the projection of horizontal angles is unaffected either by Earth curvature or by the height of the observer above the control points. (This is not true of the 'Canadian Grid' overlay which is used to measure *distances* between features in the photograph—clearly scale depends directly on the height of the observer.) However, the projection of radials is affected by the inclination of the optic axis of the camera: a change of 5° from the nominal inclination of 30° for trimetrogon obliques, introduces an error of 5 per cent in the measured angle subtended by control points near the horizontal. In the case of camera swing, both methods need to use the apparent horizon to define the vertical through the optic axis (unless there is separate information about the aircraft pitch).

Given the geographical positions of the chosen control points, it is possible to compute the angles in the horizontal plane, subtended at an assumed position. An iterative computer program has been written which

adjusts the assumed position to one having a least square difference between observed and computed angles for a number of control points. Three control points are required for a unique position; using more than three points, the probable error in the position obtained is reduced. The restriction that applies to a conventional resection, namely that the control points must not lie on a circle through the position occupied, applies to this method. Position fixes have been obtained by this method at distances as great as 100 km. from mapped surface features which were visible on oblique aerial photographs. The uncertainty of the positions obtained by this method has been found to be a few per cent of the range of the control points. Figure 3 shows a track plot using numerous fixes obtained from oblique aerial photographs.

For dead reckoning the analysis of radio echo sounding flights has mainly relied upon air data. An S.F.I.M. flight recorder has been used to record, in an analogue form, all flight parameters for dead reckoning. Pitot and static air pressure and air temperature were monitored, and from these

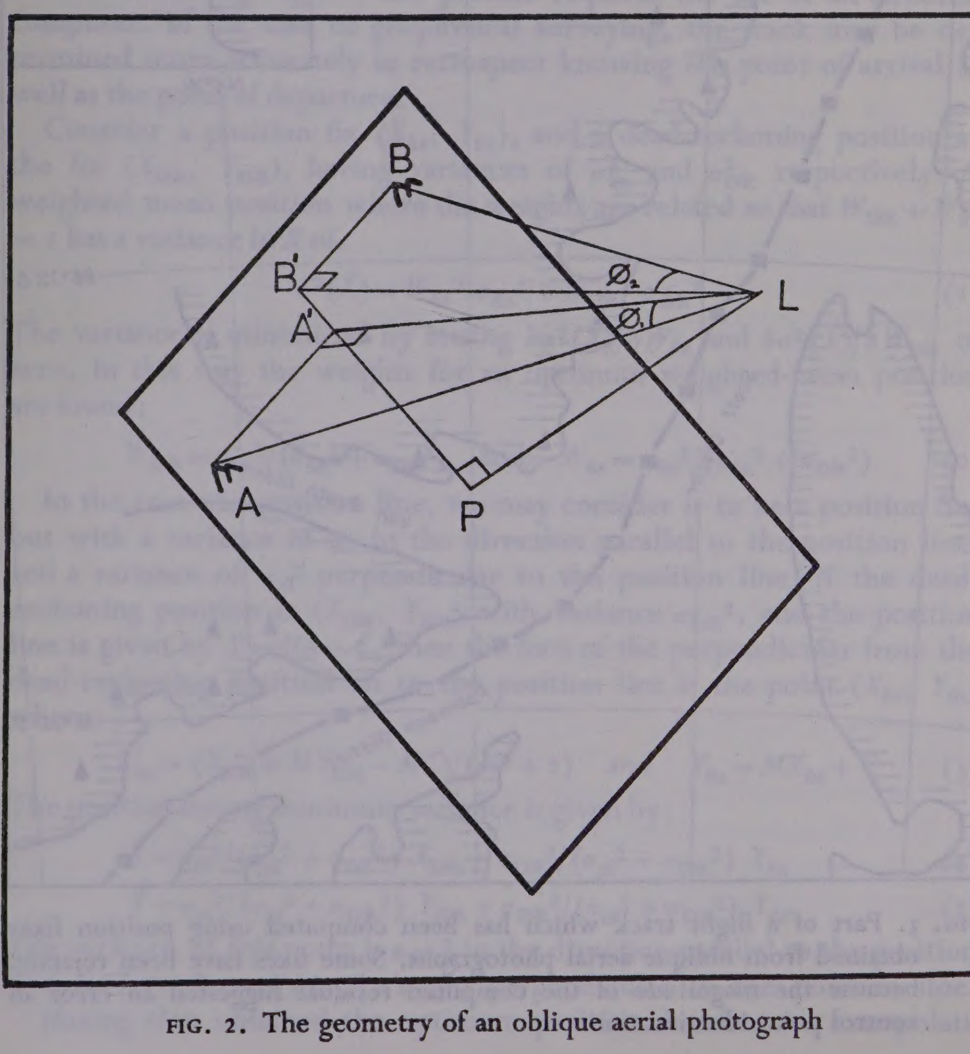

FIG. 2. The geometry of an oblique aerial photograph

true air speed was found. Heading information was provided by a free gyro on the larger aircraft, and by a gyro-stablized magnetic compass on the smaller twin Otter aircraft which was used at a greater distance from the magnetic pole. More recently a doppler system has been intermittently available on the C130 aircraft. The first results obtained from this indicate considerable improvement over the air data system. The root mean square differences between dead-reckoning position and fix positions have been about 2 metres per second of dead-reckoning time for the doppler system as opposed to about 10 metres per second for air data dead reckoning. The flight recorder trace was digitized using a

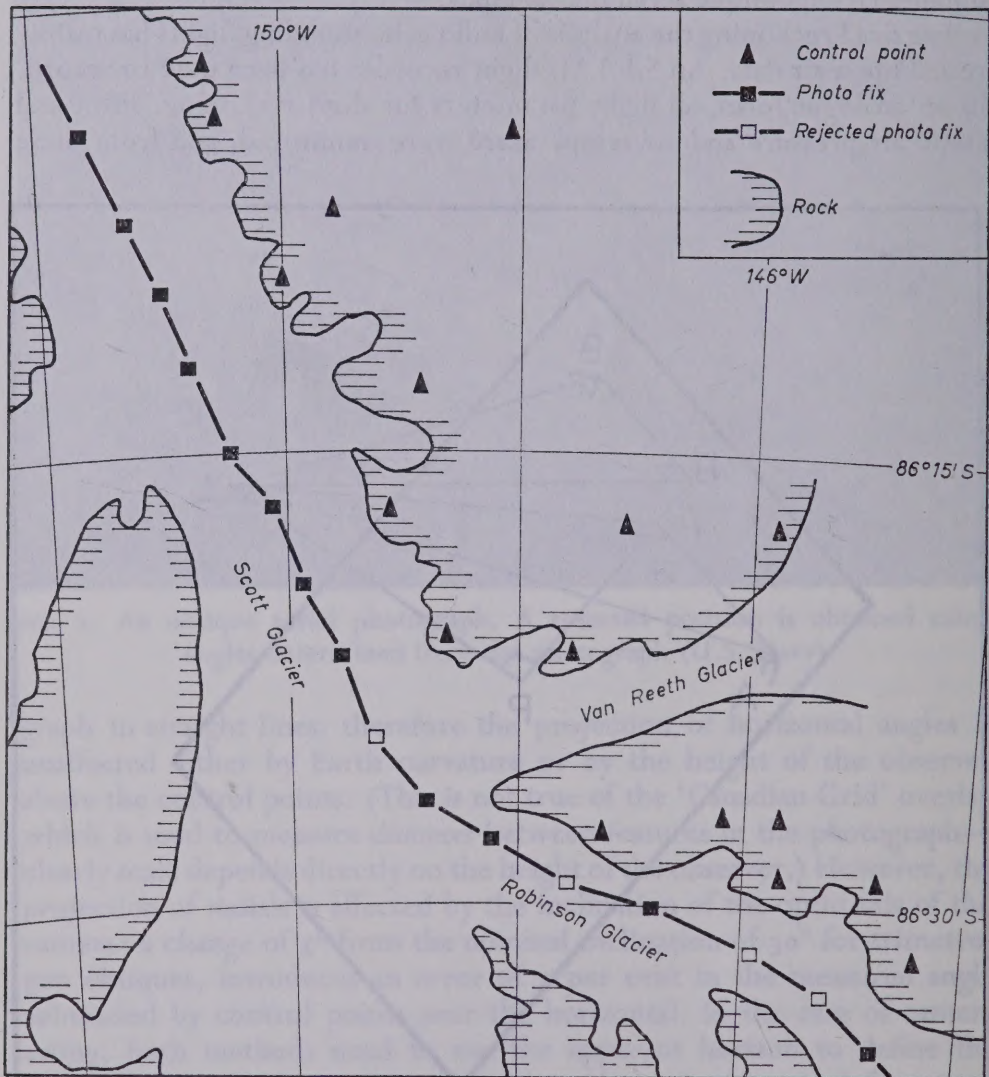

FIG. 3. Part of a flight track which has been computed using position fixes obtained from oblique aerial photographs. Some fixes have been rejected because the magnitude of the computed residual suggested an error in control point identification.

semi-automatic coordinate trace reader, and the data converted into a computer compatible paper tape form.

The larger C121 and C130 aircraft were equipped for celestial observations to allow navigation for long distances in areas devoid of surface features, and position lines were obtained by the altitude-intercept method. Position lines, as opposed to fixes, may also be obtained from oblique aerial photographs. Since the trimetrogon cameras were fixed with respect to the aircraft, the azimuth of the centre line is known, and hence the azimuth of any feature on the photograph can be computed. The incorporation of position lines into the computation of corrected track is discussed below.

3. NAVIGATIONAL ACCURACY IN GENERAL. There is a probable error in both the position-fixing system and the dead-reckoning system. The nature and magnitude of these errors have been discussed.[4] Ramsayer has proposed a method by which the important systematic errors in the dead-reckoning system are determined by least squares adjustment, and corrected for during flight[5]; the process requires the use of an airborne computer. In the case of geophysical surveying, the track may be determined more accurately in retrospect knowing the point of arrival as well as the point of departure.

Consider a position fix (X_{fix}, Y_{fix}), and a dead-reckoning position at the fix (X_{DR}, Y_{DR}), having variances of σ^2_{fix} and σ^2_{DR} respectively. A weighted mean position where the weights are related so that $W_{\mathrm{DR}} + W_{\mathrm{fix}} = 1$ has a variance in X of:

$$\sigma^2(\hat{X}) = W_{\mathrm{fix}}{}^2\,\sigma_{\mathrm{fix}}{}^2 + W_{\mathrm{DR}}{}^2\,\sigma_{\mathrm{DR}}{}^2 \tag{1}$$

The variance is minimized by setting $\delta\sigma^2(\hat{X})/\delta W_{\mathrm{fix}}$ and $\delta\sigma^2(\hat{X})/\delta W_{\mathrm{DR}}$ to zero. In this way the weights for an optimum weighted-mean position are found:

$$W_{\mathrm{DR}} = \sigma_{\mathrm{fix}}{}^2/(\sigma_{\mathrm{fix}}{}^2 + \sigma_{\mathrm{DR}}{}^2) \quad \text{and} \quad W_{\mathrm{fix}} = \sigma_{\mathrm{fix}}{}^2/(\sigma_{\mathrm{fix}}{}^2 + \sigma_{\mathrm{DR}}{}^2) \tag{2}$$

In the case of a position line, we may consider it to be a position fix, but with a variance of ∞ in the direction parallel to the position line, and a variance of $\sigma_{\mathrm{pl}}{}^2$ perpendicular to the position line. If the dead-reckoning position is (X_{DR}, Y_{DR}) with variance $\sigma_{\mathrm{DR}}{}^2$, and the position line is given by $Y = MX + C$, then the foot of the perpendicular from the dead-reckoning position on to the position line is the point (X_{fix}, Y_{fix}) where:

$$X_{\mathrm{fix}} = (X_{\mathrm{DR}} + MY_{\mathrm{DR}} - MC)/(M^2 + 1) \quad \text{and} \quad Y_{\mathrm{fix}} = MX_{\mathrm{fix}} + C \tag{3}$$

The position having minimum variance is given by:

$$\hat{X} = \sigma_{\mathrm{pl}}{}^2/(\sigma_{\mathrm{pl}}{}^2 + \sigma_{\mathrm{DR}}{}^2)\; X_{\mathrm{DR}} + \sigma_{\mathrm{DR}}{}^2/(\sigma_{\mathrm{pl}}{}^2 + \sigma_{\mathrm{DR}}{}^2)\; X_{\mathrm{fix}} \tag{4}$$

$$\hat{Y} = \sigma_{\mathrm{pl}}{}^2/(\sigma_{\mathrm{pl}}{}^2 + \sigma_{\mathrm{DR}}{}^2)\; Y_{\mathrm{DR}} + \sigma_{\mathrm{DR}}{}^2/(\sigma_{\mathrm{pl}}{}^2 + \sigma_{\mathrm{DR}}{}^2)\; Y_{\mathrm{fix}} \tag{5}$$

The variance of this point is $\sigma_{\mathrm{DR}}{}^2$ in the direction parallel to the position line, and $(\sigma_{\mathrm{pl}}{}^2\ \sigma_{\mathrm{DR}}{}^2)/(\sigma_{\mathrm{pl}}{}^2 + \sigma_{\mathrm{DR}}{}^2)$ perpendicular to the position line.

Having thus obtained the optimum position at a fix, we extrapolate

the position to future time by means of a dead-reckoning system. In general the variance of the dead-reckoning system degrades with time. However, unlike the in-flight case dealt with by Ramsayer,[5] the point of arrival at a new fix may be used to improve a post-flight track. At each new fix a similar optimum position may be found by application of the above expressions.

We will now consider random errors in the dead-reckoning system and neglect for the moment errors in the fixes. Suppose that the departure of the dead-reckoning track from the actual track is formally the same as a Brownian motion in both directions X and Y. This will be the case if the dead-reckoning system integrates velocity signals having gaussian noise but no systematic error on them. If the root mean square velocity of departure in one dimension is V_{rms}, the probability distribution of departure x after a time t is normal with variance $V_{\text{rms}}^2 t$.

$$P(x, t) = \frac{1}{V_{\text{rms}} \sqrt{(2\pi t)}} \exp\left[\frac{-x^2}{2 V_{\text{rms}}^2 t}\right] \tag{6}$$

If at some time t the departure is known to be x, then at some later time T the departure X will be distributed according to:

$$P(X, T/x, t) = \frac{1}{V_{\text{rms}} \sqrt{\{2\pi(T-t)\}}} \exp\left[\frac{-(X-x)^2}{2V_{\text{rms}}(T-t)}\right] \tag{7}$$

It may be shown that the probability distribution of departure x after time t, given the departure X after time T, is: $P(x, t/X, T) = P(X, T/x, t).P(x, t)/P(X, T)$. Hence knowing the departure X after time T by means of a position fixing system, the departure x after some lesser time t is distributed thus:

$$P(x, t/X, T) = \frac{1}{V_{\text{rms}} \sqrt{\left\{2\pi t\left(1-\frac{t}{T}\right)\right\}}} \exp\left[\frac{-\left(x-\frac{t}{T}X\right)^2}{2 V_{\text{rms}} t\left(1-\frac{t}{T}\right)}\right] \tag{8}$$

This is a normal distribution centred on $(t/T)X$ with standard deviation $V_{\text{rms}}\sqrt{\{t(1-t/T)\}}$. Hence we have obtained the well-known result that the probable error in the track can be minimized by making $\{x-(t/T)x\}$ the corrected track. The root mean square value of X is $V_{\text{rms}}\sqrt{(T)}$ and the ratio of x_{rms} to X_{rms} is $\sqrt{(t/T - t^2/T^2)}$. This has a maximum value (when $t = T/2$) of $0{\cdot}5$. Thus the adjusted track may be in error by half the root mean square observed departure at a new fix.

We will now consider the effect of certain systematic errors in the dead-reckoning system, and again neglect errors in the fixes. Define the following vector positions:

- $\mathbf{A}(t)$ The actual track between ideal fixes
- $\mathbf{B}(t)$ The track obtained by dead reckoning
- $\mathbf{C}(t)$ The corrected track

and the following vector velocities:

$\mathbf{V}(t)$ The true velocity vector
$\mathbf{W}(t)$ The velocity vector used by the dead-reckoning system

For simplicity in assessing these errors assume that navigation is in a plane. If at time $t=0$ the vector position is known to be $\mathbf{P}$, then we can write the expressions:

$$\mathbf{A}(t)=\mathbf{P}+\int_0^t \mathbf{V}(\tau)\,d\tau \quad\text{and}\quad \mathbf{B}(t)=\mathbf{P}+\int_0^t \mathbf{W}(\tau)\,d\tau \tag{9}$$

Then the departure of the dead-reckoning track from the true track is:

$$\mathbf{B}(t)-\mathbf{A}(t)=\int_0^t \mathbf{W}(\tau)-\mathbf{V}(\tau)\,d\tau \tag{10}$$

If at time T the true vector position is known, an ideal fix, then the value of $[\mathbf{B}(T)-\mathbf{A}(T)]$ is observed and the corrected track is obtained by adjusting the dead-reckoning track in accordance with the rule derived for a random departure, i.e. linearly with time, $\mathbf{C}(t)=(\mathbf{B}t)-(t/T)\,[\mathbf{B}(T)-\mathbf{A}(T)]$ which is:

$$\mathbf{C}(t)=\mathbf{P}+\int_0^t \mathbf{W}(\tau)\,d\tau-(t/T)\int_0^T [\mathbf{W}(\tau)-\mathbf{V}(\tau)\,d\tau] \tag{11}$$

From this we obtain the difference between the true track and the corrected track:

$$\mathbf{C}(t)-\mathbf{A}(t)=\int_0^t [\mathbf{W}(\tau)-\mathbf{V}(\tau)]\,d\tau-(r/T)\int_0^T [\mathbf{W}(\tau)-\mathbf{V}(\tau)]\,d\tau \tag{12}$$

It may be seen that this is identically zero at times $t=0$ and $t=T$, that is the corrected track is constrained to pass through the initial and final fixes regardless of any error in the dead-reckoning system. At other times, the difference between the corrected track and the true track depends upon $\mathbf{W}(t)-\mathbf{V}(t)$, that is on errors in the dead-reckoning system.

The propagation of errors in doppler and inertial navigators used as 'open loop' dead-reckoning systems is a much-studied subject. We will substitute some likely systematic errors in these systems into Eqn. (12) above, and evaluate their effect on the corrected track.

(1) A systematic error in the scale factor of velocity might be the result of miscalibration of the dead-reckoning system or, for example, misalignment of a doppler beam. We may then write: $\mathbf{W}(t)=(1+\delta)\,\mathbf{V}(t)$. Equation (10) above reduces to:

$$\mathbf{C}(t)-\mathbf{A}(t)=\int_0^t \mathbf{V}(\tau)\,d\tau-(t/T)\int_0^T \delta\mathbf{V}(\tau)\,d\tau \tag{13}$$

This is only zero (the corrected track identical to the true track) if $\int_0^t \mathbf{V}(\tau)\,d\tau=\alpha t$ which is true if $\mathbf{V}(t)$ is a constant. Otherwise the error in the corrected track at any instant is equal to δ multiplied by the distance between the actual position and the position interpolated along a straight line between the fixes. Clearly the more nearly straight the actual track, the more accurate the corrected track.

(2) A systematic velocity offset might be due to miscalibration of the

dead-reckoning system or to some bias in a doppler navigator frequency tracker. Note that a constant wind vector is formally the same as this in the case of a dead-reckoning system which relies upon air data. We may then write: $\mathbf{W}(t)=\mathbf{K}(t)+\mathbf{V}(t)$. Equation (12) then becomes:

$$\mathbf{C}(t)-\mathbf{A}(t)=\int_0^t \mathbf{K}(\tau)\,d\tau-(t/T)\int_0^T \mathbf{K}(\tau)\,d\tau \tag{14}$$

which for a constant $\mathbf{K}(t)$ is identically zero. Thus a constant error in velocity does not cause an error in the corrected track. If the velocity offset is due to some bias in the frequency tracker of a doppler navigator, then $\mathbf{K}(t)$ is only constant if $\mathbf{V}(t)$ is also constant. If $\mathbf{V}(t)$ is not constant then $\mathbf{K}(t)$ will vary and the corrected track will be in error. In the case of an air data dead-reckoning system, the wind vector $\mathbf{K}(t)$ is most likely not to be constant and again the corrected track is in error.

(3) A constant error in the heading information may be caused by incorrect alignment of the compass gyro or, in the case of a magnetic compass, by incorrect magnetic variation information. In this case if we write $\mathbf{W}(t)=V'\exp[j\theta]$ and $\mathbf{V}(t)=V'\exp[j\phi]$, where V' is a scalar speed and the heading error is equal to $(\theta-\phi)$, Eqn (12) reduces to

$$\mathbf{C}(t)-\mathbf{A}(t)=[\exp(j\delta)-1]\left\{\int_0^t \mathbf{V}(\tau)\,d\tau-(t/T)\int_0^T \mathbf{V}(\tau)\,d\tau\right\} \tag{15}$$

Once again we see that if $\mathbf{V}(t)$ is constant this is identically zero and the corrected track is the true track.

(4) A constant rate of change of heading error is not infrequently a property of an imperfect gyro. In this case we write: $\mathbf{W}(t)=V'\exp[j(\phi+\delta t)]$ and $\mathbf{V}(t)=V'\exp[j\phi]$. If $\mathbf{V}(t)$ is a constant, then equation (12) becomes:

$$\frac{\mathbf{V}e^{j\phi}}{j\delta}\left\{(e^{j\delta t}-1)-\frac{t}{T}(e^{j\delta T}-1)\right\} \tag{16}$$

If δT is small this reduces to $\mathbf{V}e^{j\phi}\,j\delta t\,(T-t)/2$. The magnitude of the difference is an extremum when: $t=T/2$, the same as for the other systematic errors, and it is equal to $V\delta T^2/8$. For example consider a gyro drift of 0·01 radians per hour. If the time between fixes is 1 hour and the distance covered in an hour is 500 km., the maximum error in the corrected track would be 0·6 km. It is clear from the above illustrations that the corrected track most closely approximates to the true track if the velocity vector is constant: that is the aircraft flies at a constant speed in a constant direction between fixes.

To illustrate the accuracy of a corrected track, the process of fitting the dead-reckoning track to the fixes has been carried out on a flight in the Antarctic Peninsula (Fig. 4). The portion of the flight shown covers about 400 km. and has been fitted to four fixes. Four other fixes which were not used to correct the track are shown together with the corresponding dead-reckoning position. Air data were used to compute the dead-reckoning track and it may be seen that the track has a number of direction changes. On a portion between fixes of about 200 km. the

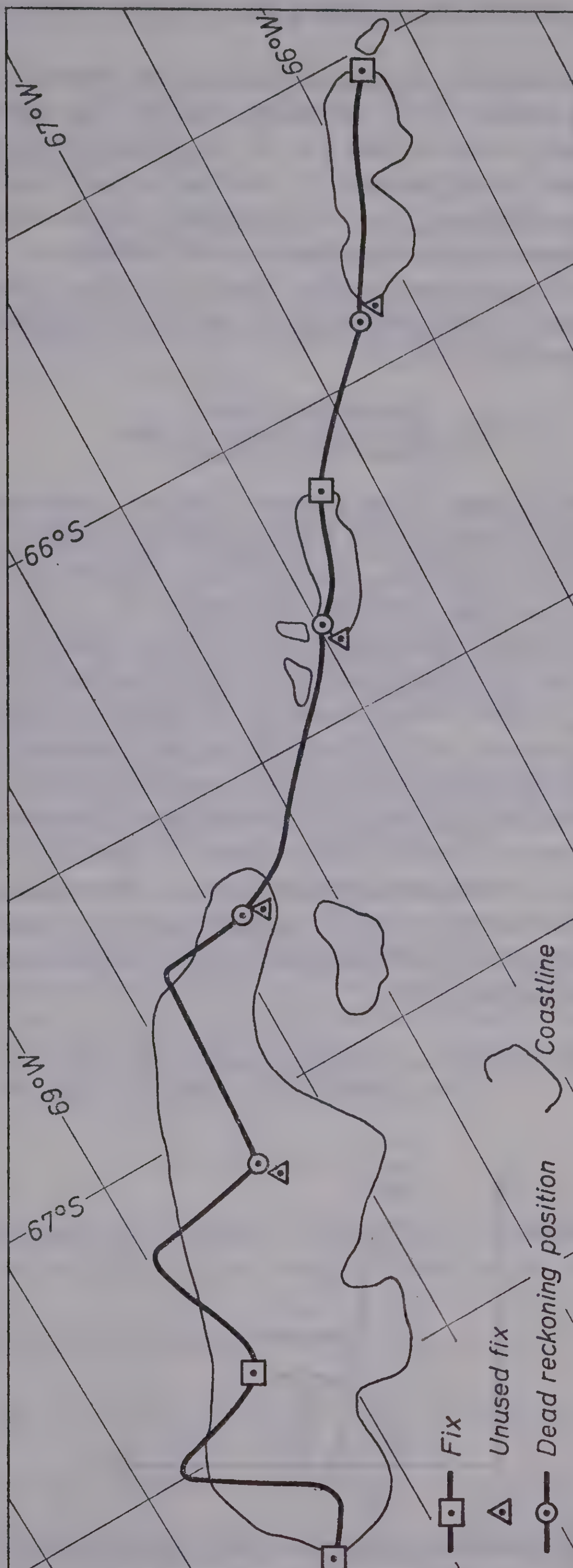

FIG. 4. Part of a flight track over the west coast of the Antarctic Peninsula. Some fixes which were not used in the computation are shown relative to the positions obtained for the corrected track.

average error at the unused fixes is about 4 km. or 2 per cent of the interfix distance.

4. INFORMATION GAINED. In general, neither the measured property $Z(X, Y)$ nor the position (X, Y) is known exactly. The information J gained in a particular measurement (Z, X, Y) is defined as the binary logarithm of the ratio of the probability that the property has the measured value after measurement, to the probability before measurement. We suppose throughout this section that it is the relation of Z to (X, Y) which is desired. As suggested earlier there are some investigations where the relationship of two properties Z_1 and Z_2 is sufficient in itself without precise knowledge of (X, Y).

$$J = \log_2 \frac{[\text{Probability after}]}{[\text{Probability before}]} \text{ bits} \tag{17}$$

The probability distribution of the property 'before' measurement may be estimated from measurements of Z along the track without reference to the absolute (X, Y) position, provided that relative positions are known accurately and the statistical properties of Z are isotropic. Alternatively some *a priori* assumption may be made about the statistical properties of Z. The probability of the property having the measured value, call it Z_m, before measurement is therefore estimated. After measurement the property is distributed about the measured value, and the width of the distribution is related to the resolution of the measuring instrument and the accuracy of positional information. Suppose that the Z, in so far as it is known before measurement, is normally distributed about a mean value $\langle Z\rangle$ with standard deviation σ_1. After measurement Z has a probability distribution which is normal about the observed value Z_m with standard deviation σ_2 as illustrated in Fig. 5. Before measurement the probability distribution of Z is:

$$P(Z)\, dZ = \frac{1}{\sigma_1 \sqrt{(2\pi)}} \exp\left[\frac{-(Z-\langle Z\rangle)^2}{2\sigma_1{}^2}\right] \tag{18}$$

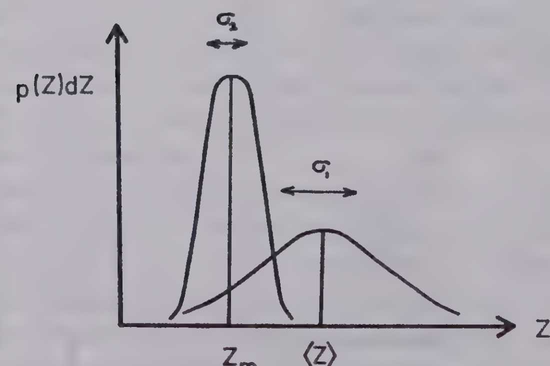

FIG. 5. The probability distribution of Z value before (σ_1) and after (σ_2) measurement.

After measurement the probability distribution of Z is:

$$P(Z)\,dZ=\frac{1}{\sigma_2\sqrt{(2\pi)}}\exp\left[\frac{-(Z-Z_{\mathrm{m}})^2}{2\sigma_2{}^2}\right] \tag{19}$$

Then before measurement

$$P(Z=Z_{\mathrm{m}})\,dZ=\frac{1}{\sigma_1\sqrt{(2\pi)}}\exp\left[\frac{-(Z_{\mathrm{m}}-\langle Z\rangle)^2}{2\sigma_1{}^2}\right] \tag{20}$$

and after measurement $P(Z=Z_{\mathrm{m}})\,dZ=1/\sigma_2\sqrt{(2\pi)}$.

The new information contained in the statement 'Z has the value Z_{m} at the point (X, Y)' is given by:

$$J=\log_2\left\{\frac{\sigma_1}{\sigma_2}\exp\left[\frac{-(Z_{\mathrm{m}}-\langle Z\rangle)^2}{2\sigma_1{}^2}\right]\right\} \tag{21}$$

This applies only to a single such statement. If a number of statements are made for points which are close together then the probability distribution 'before' a given statement is made will be modified by the data already given, that is to say σ_1 is reduced and the information gained is reduced.

Let us make the additional assumption that the function A has a normalized auto-correlation function which is gaussian: $C(\tau)=\exp[-\tau^2/T^2]$ where τ is the separation in the X, Y plane.

Then it may be shown[6] that the mean square gradient is $\langle\psi^2\rangle=\beta_0{}^2/2$, where $\tan\beta_0=2\sigma_1/T$. It may be shown further that instrument resolution σ_{res} and position error σ_{p} are combined thus:

$$\sigma_2{}^2=\sigma_{\mathrm{res}}{}^2+(\sigma_{\mathrm{p}}+\langle\xi\rangle^2)\langle\psi^2\rangle \tag{22}$$

where $\langle\xi\rangle$ is the mean value of the position error.

If the mean error in position is zero, that is to say there are no systematic errors in the corrected position which is used, then the expression becomes $\sigma_2{}^2=\sigma_{\mathrm{res}}{}^2+\sigma_{\mathrm{p}}{}^2\langle\psi^2\rangle$, and for small mean square gradient we obtain the important result:

$$\sigma_2{}^2=\sigma_{\mathrm{res}}{}^2+2\sigma_{\mathrm{p}}{}^2\frac{\sigma_1{}^2}{T^2} \tag{23}$$

thus formalizing the intuitive result that to maximize the information gathered we require accurate navigation and precise measurement of the property.

The information J, gained in a single measurement, is then:

$$J=\log_2\left\{\sigma_1\exp\left[\frac{-(Z_{\mathrm{m}}-\langle Z\rangle)^2}{2\sigma_1{}^2}\right]\right\}-\tfrac{1}{2}\log_2\left[\sigma_{\mathrm{res}}{}^2+\frac{2\sigma_{\mathrm{p}}{}^2\sigma_1{}^2}{T^2}\right] \tag{24}$$

As a function of instrument resolution, or positional error, J has the form $J=A-B\log(C+x)$ where A, B and C are constants and x is σ_{res} or σ_{p}: this is illustrated in Fig. 6. The knee of the curve occurs when the value of J is reduced equally by the sensor resolution and by the naviga-

tional accuracy, which corresponds to $\sigma_{res} = \sqrt{(2)}\ \sigma_p\ \sigma_1/T$. It is clear from the diagram that improving the sensor resolution beyond $\sigma_{res} < \sqrt{(2)}\sigma_p\ \sigma_1/T$ does not increase the quantity of information gained and conversely it is unprofitable to improve the navigational accuracy more than $\sigma_p < \sigma_{res} T/\sqrt{(2)}\ \sigma_1$.

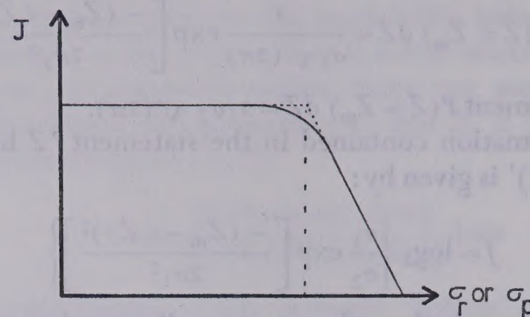

FIG. 6. The information gained (J) as a function of uncertainty in position (σ_p) or measurement (σ_{res})

As an example, consider the measurement of ice thickness to a resolution of 10 m. The X, Y auto-correlation distance depends on the area in question, but on a floating ice shelf it may be as much as 50 km. The standard deviation of thickness in such an area is perhaps 100 m. The critical figure for navigational accuracy is then about 3·5 km. in position. However, in an area where the ice is disturbed by underlying rock the auto-correlation distance would be much less, and the demands made upon navigation correspondingly greater. In particular an area where the subglacial topography is mountainous may have an auto-correlation distance of only 5 km. and the standard deviation of thickness may be 500 m. In this case the critical navigational accuracy would be about 70 m. If the navigational accuracy were worse than this figure, the quantity of information gained would be accordingly less. For example, if positions were accurate to about 1 km., this would correspond to a resolution in thickness of about 150 m. which is considerably worse than the limit imposed by the sensor resolution.

The ice surface of the inland Antarctic plateau is remarkably smooth. Measurements conducted during a radio echo sounding flight in latitude 130° E. longitude 73 °S. indicate an auto-correlation distance of 500 km. The uncertainty in elevation measurement is at least 100 m., using aneroid altimetry with little control, and the standard deviation of the height distribution was 1000 m. These figures indicate that to conduct surface altimetry by this technique the critical uncertainty in position is 20 km.

These same general relations can be applied to depth, magnetic or other profiling from a ship. Consider the measurement of magnetic anomalies with a resolution of 5 gamma. A typical auto-correlation

distance is 20 km., and the standard deviation of the variations is 500 gamma. In this case the critical navigational uncertainty is about 150 m. A shipborne range-rate satellite navigator achieves an accuracy of about 1000 m.[4]; this means that the effective resolution in the magnetic measurement is no better than 35 gamma. An advanced satellite navigator using doppler and inertial determination of ship velocity, achieves the critical accuracy of 150 m.,[7] and in this case the effective resolution reduces to about 7 gamma.

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